Mathematical modelling of tumor growth using a compressible Navier-Stokes-Cahn-Hilliard model.

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The Cahn-Hilliard (CH) equation [1] is a fourth-order parabolic model, originally introduced to represent spinodal decomposition in binary alloys under a sudden cooling. Fifteen years ago [3], this model found a new application: the modelling of tumor growth. Nowadays, many variants of the CH model exist to simulate the growth of a tumor and include more and more physical or biological effects. All these extensions introduce new challenges: for the analysis, the numerical simulations and also to connect the results to the biology.

During this presentation, we aim to motivate and explain a new general Cahn-Hilliard model that comprises a lot of different effects: attraction and repulsion effects between the cells, viscosity of the tumorous and healthy tissues, friction of the cells on the extra-cellular fibers and motility of the cells. These three latter effects are of main importance as a contrast of mechanical properties between the tumor and the healthy tissue can result in the emergence of finger-like protrusions as the tumor grows. This phenomena is important to study as the presence of irregular protrusions is a marker of bad prognosis.

Our model couples the CH and the compressible Navier-Stokes equations [2]. We will present some elements of its analysis and describe a novel numerical scheme that preserves the properties of the solutions. We will then present a series of numerical simulations that highlights the properties of the scheme and depicts the solutions of the system in time.

References

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